Some Selected Picks from Extremal Combinatorics

Sam Spiro, UC San Diego.

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For example, a central problem of Ramsey Theory is to determine how large a clique K_N one can have before every two-coloring of its edges contains a monochromatic clique of size t. I.e., it seeks to determine how many "vertices" a "two-colored cliques K_N without a monochromatic clique of size t" can have.

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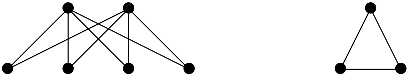

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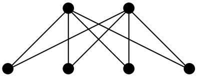

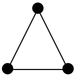

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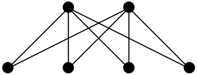

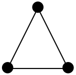

Question

How many "edges" can an "n-vertex F-free graph" have?

We denote this quantity by $\text{ex}(n, F)$, which is called the extremal number or Turán number of F.

Extremal Graph Theory

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For example, $\mathrm{ex}(n, K_3) \geq \lfloor n^2/4 \rfloor$ because $G = K_{\lfloor n/2 \rfloor, \lceil n/2 \rceil}$ is an n-vertex graph which is K_3-free and with $\lfloor n^2/4 \rfloor$ edges.

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Theorem (Mantel, 1907)

$$\mathrm{ex}(n, K_3) = \lfloor n^2/4 \rfloor,$$

and the unique construction is $K_{\lfloor n/2 \rfloor, \lceil n/2 \rceil}$.

Extremal Graph Theory

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Theorem (Turán, 1941)

$$\mathrm{ex}(n, K_t) = \left\lfloor \left(1 - \frac{1}{t-1}\right) \frac{n^2}{2} \right\rfloor,$$

and the unique construction is a complete $(t-1)$*-partite graph with parts of sizes* $\lfloor n/(t-1) \rfloor, \lceil n/(t-1) \rceil$.

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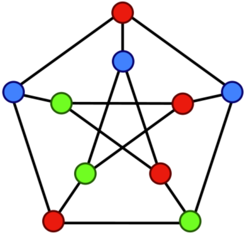
For example, the following shows that the Petersen graph G has $\chi(G) \leq 3$, and it is not hard to show that it is in fact equal to 3.



Picture form Wikipedia.

Extremal Graph Theory

Theorem (Erdős-Stone, 1946)

For any graph F,

$$\mathrm{ex}(n, F) = \left(1 - \frac{1}{\chi(F) - 1}\right) \frac{n^2}{2} + o(n^2).$$

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Note that when $\chi(F) = 2$, this only gives

$$\mathrm{ex}(n, F) = o(n^2).$$

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In general determining $\mathrm{ex}(n, F)$ when F is bipartite (i.e. $\chi(F) = 2$) is very, very difficult, even for simple families of graphs.

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Conjecture (Erdős-Sós, 1962)

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Theorem (Kovari-Sós-Turán, 1954)

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This is known to hold whenever $t > (s-1)!$ due to the existence of projective norm graphs.

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In general, it is often the case that a conjectured upper bound is relatively easy to obtain, but seemingly the lower bound requires tools from algebra, geometry, number theory, etc.

Hypergraphs

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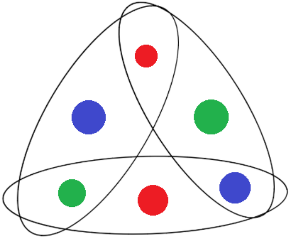

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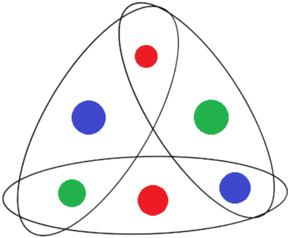

When each $e \in E$ has r vertices we say that H is an r-uniform hypergraph.

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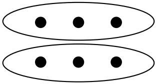
In general, many problems which are easy for graphs become much harder for hypergraphs.

Hypergraphs

Let $M_2^{(r)}$ be two disjoint r-sets.



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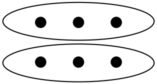

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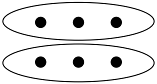

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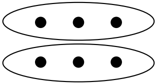

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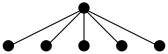

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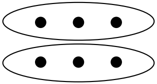

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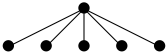

and in general the hypergraph $S_v^{(r)}$ is called a star.

Hypergraphs

Theorem (Erdős-Ko-Rado, 1961)

If $n \geq 2r$ then

$$\operatorname{ex}(n, M_2^{(r)}) = \binom{n-1}{r-1},$$

and for $n > 2r$ the unique construction is the star.

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A natural generalization of EKR is to ask for the extremal number of $M_s^{(r)}$, which is the disjoint union of s distinct r-sets.

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Conjecture (Erdős Matching Conjecture, 1965)

If $n \geq sr - 1$, *then*

$$\mathrm{ex}(n, M_s^{(r)}) = \max\left\{\binom{n}{r} - \binom{n-s+1}{r}, \binom{sr-1}{r}\right\}.$$

We say that a set of k integers is a k-term arithmetic progression (or simple a k-AP) if they are of the form $\{a, a + d, a + 2d, \ldots, a + (k - 1)d\}$.

Combinatorial Number Theory

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Question

How many "integers in $\{1, \dots, n\}$" can "a set A not containing any k-AP" have?

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How many "integers in $\{1, \ldots, n\}$" can "a set A not containing any k-AP" have?

Let us adopt the (non-standard) notation $\mathrm{ex}(n, k\text{-}\mathrm{AP})$ for the size of a largest k-AP free subset of $\{1, \ldots, n\}$.

Combinatorial Number Theory

Theorem (Behrend, 1946)

There exists a constant $c > 0$ *so that for* $k \geq 3$,

$$\mathrm{ex}(n, k\text{-}AP) \geq e^{-c\sqrt{\log n}} \cdot n.$$

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For all k,

$$\mathrm{ex}(n, k\text{-}AP) = o(n).$$

That is, every dense set of integers contains arbitrarily long arithmetic progressions.

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Theorem (Schoen, May 2020)

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Theorem (Schoen, May 2020)

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This upper bound is roughly $e^{-\log\log n} \cdot n$, while the best lower bound is still $e^{-c\sqrt{\log n}} \cdot n$, and closing this a gap is a huge problem in the field.

Erdős Numbers

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As you may have realized, Erdős has written a lot of papers (both in extremal combinatorics and in general).

This has led to the amusing statistic: Erdős has Erdős number 0. If you write a paper with someone who has Erdős number k, then your Erdős number is $k + 1$.

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MR Erdos Number = 4			
Claus Mazanti Sorensen	coauthored with	Hui Gao[6]	MR3532238
Hui Gao[6]	coauthored with	Tong Liu[2]	MR3263170
Tong Liu[2]	coauthored with	Wen-Ching Winnie Li	MR3105749
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It tends to be even smaller once you start picking extremal combinatorialists!

MR Erdos Number = 2			
Sam Spiro	coauthored with	Fan Chung	MR4057523
Fan Chung	coauthored with	Paul Erdős[1]	MR0889356

Wrapping Things Up

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Rapping Things Up

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Bear in mind for this R A P,

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Bear in mind for this R A P,
I got a Behrend mind and no K A P.

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You won't be progressing when I'm the MC,

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You won't be progressing when I'm the MC,
Got enough phat disses for a PhD.

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When we spar you'll be breathless, needing CPR,

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You won't be progressing when I'm the MC,
Got enough phat disses for a PhD.

When we spar you'll be breathless, needing CPR,
Seeing stars when I hit you with that EKR.

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Am I getting my point across, should I spell it out for you?

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Am I getting my point across, should I spell it out for you?
AMGM, all your products are less than my average, μ.

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Was that too intense, were you only semi-ready?

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You're acting so dense I could apply Szemeredi.

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I'm not the kind of person you should turn on,

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I'm not the kind of person you should turn on,
Got edge to the max, like I'm Turan.

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I'm an extremal guy, Erdos-honed,

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My level's so high it's like I'm Erdős-Stoned.

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Getting 10 out of 10's Fall, Winter, Spring and Summer,

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Getting 10 out of 10's Fall, Winter, Spring and Summer,
Your only 10 is your Erdős number.

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Now I'd rather be nice, stay home and count,

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Now I'd rather be nice, stay home and count,
But if you want to fight, you're going down for the count.

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Now I'd rather be nice, stay home and count,
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Bring your whole clique, I'll bring my acrobatics,

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Now I'd rather be nice, stay home and count,
But if you want to fight, you're going down for the count.

Bring your whole clique, I'll bring my acrobatics,
Color you black and blue until you're monochromatic.

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Now I'd rather be nice, stay home and count,
But if you want to fight, you're going down for the count.

Bring your whole clique, I'll bring my acrobatics,
Color you black and blue until you're monochromatic.

Keep coming and coming, there's no need for dramatics,

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Keep coming and coming, there's no need for dramatics,
I'll keep cutting and cutting, chop you into dyadics.

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Don't think I'm care-free just cause I'm doing mathematics,

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Don't think I'm care-free just cause I'm doing mathematics,
If you mess with me, things will get hyper graphic.

The End

(Drops Mic)

Answers to Questions

1. Yes, I did make the rap first and then designed the talk around explaining it.

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1. Yes, I did make the rap first and then designed the talk around explaining it.
2. Yes, I am open to performing at weddings as well as bar/bat mitzvahs.

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1. Yes, I did make the rap first and then designed the talk around explaining it.
2. Yes, I am open to performing at weddings as well as bar/bat mitzvahs.
3. My (tentative) rapper name is “Lil Oh.”